We did not examine the case of a bubble which is simultaneously undergoing pulsations and translation. Thus, Kelvin–Helmholtz instability of the bubble surface (connected with discontinuity of tangential velocity at the gas–liquid boundary) remains outside the scope of the present discussion. However, it can be suggested that, given sufficiently small initial translation velocities of a bubble, Kelvin–Helmholtz instability will be less important than Taylor instability and the conclusions reached here will remain valid.

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STUDY OF NONSTEADY LOADS IN THE ACCELERATED AND SUDDEN MOTION OF BODIES OF DIFFERENT FORM

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Together with the need to calculate the aerodynamic and strength characteristics of bodies during steady-state motion, it is often necessary to evaluate nonsteady forces acting during abrupt changes in the velocity regime – especially during sudden acceleration of a body from a state of rest to a specified steady flight velocity. It is interesting to determine the additional loads (compared to the steady phase of motion) that develop during nonsteady flow past the body. Here, the important characteristics are the maximum possible pressure and force and the characteristic time of the nonsteady transitional processes.

Below we examine the problem of the accelerated motion of certain bodies (a sphere, a cylinder with a flat edge, and a cone) from a state of rest to a specified subsonic or supersonic velocity with different accelerations. We will include the case of sudden motion of the body with a prescribed velocity. Using a numerical method, we obtain the nonsteady aerodynamic characteristics of the body for different accelerations. An analytical method is proposed for calculating the pressure distribution at the initial moment of time and the maximum forces present in the case of sudden motion.

1. Formulation of the Problem and Method of Numerical Solution. Let a solid of revolution of a specified form begin to move from a state of rest at the initial moment of time $t = 0$. Moving with steady acceleraton during the time T, the body is assumed to reach a velocity corresponding to a prescribed Mach number M. The gas is considered to be ideal and to be in a state of rest with a constant pressure p_0 and density ρ_0. The adiabatic exponent of the gas is $\gamma = 1.4$.

In the coordinate system connected with the body, the flow of the gas is described by the two-dimensional nonsteady Euler equations

$$\frac{\partial}{\partial t}(\rho y) + \frac{\partial}{\partial x}(\rho u y) + \frac{\partial}{\partial y}(\rho v y) = 0,$$

$$\frac{\partial}{\partial t}(\rho u y) + \frac{\partial}{\partial x}[(p + \rho u^2) y] + \frac{\partial}{\partial y}(\rho u v y) = 0,$$

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$$\frac{\partial}{\partial t}(\rho v y) + \frac{\partial}{\partial x}(\rho u v y) + \frac{\partial}{\partial y}[(p + \rho v^2) y] = p,$$

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{u^2 + v^2}{2}\right) y\right] + \frac{\partial}{\partial x}\left[\rho u\left(e + \frac{p}{\rho} + \frac{u^2 + v^2}{2}\right) y\right] + \frac{\partial}{\partial y}\left[\rho v\left(e + \frac{p}{\rho} + \frac{u^2 + v^2}{2}\right) y\right] = 0$$

and the equation of state $p = \rho e(\gamma - 1)$. Here, p is pressure; ρ, density; u and v, longitudinal and vertical components of velocity in the Cartesian coordinate system x, y; e, specific energy of a unit mass of gas; t, time.

We take the following as the characteristic dimensional values: the dimension of the body L, the unperturbed values of pressure p_0 and density ρ_0, velocity $u_0 = \sqrt{p_0/\rho_0}$, time $t_0 = L/u_0$, and force $F = p_0 L^2$. The quantity L is the radius of the sphere, the base of the cylinder, or the base of the cone. All of the results will be given in dimensionless form.

We will solve the problem by the finite difference scheme of Godunov [1], using a modified program for study of two-dimensional nonsteady flow about bodies of different form [2, 3]. We assign conditions of impermeability as the boundary conditions on the body and on the symmetry axis. The initial conditions correspond to the state of rest.

The problem is solved on a computational grid consisting of N rays emanating from the coordinate origin and K concentric lines that repeat the form of the body ($N \times K = 135 \times 51$). For example, for a sphere these are concentric circles approximately replaced by broken lines. The subdivision of the grid is uniform with respect to the angle and nonuniform over the radius. In the case of the radius, the step size increases in a geometric progression toward the outside boundary of the grid. This boundary is located about 10-12 characteristic dimensions from the center of the body.

2. Results of Numerical Calculations. We performed a series of calculations for the problem of the acceleration of two bodies — a sphere and a semiinfinite cylinder with its end turned toward the flow. Here we used different accelerations corresponding to the time of acceleration T = 0.1-1.8. Figure 1 shows the time dependence (solid curves) of the nonsteady total force for a sphere of unit radius with different values of T and M = 2. With an increase in acceleration, the maximum increases and approaches the limiting value F_S (at T = 0). This corresponds to instantaneous motion with M = 2.

It should be noted that even for relatively slow accelerations (T = 1-2; T ≈ 1 is the time of propagation of perturbations over ~1.2 radii of the body) there is appreciable "overshooting" of the force compared to the steady-state value (by about 20%). In the case of sudden motion, this overshooting reaches 80%. The dashed line in Fig. 1 corresponds to the moment of time when the instantaneous velocity of the body reaches M = 1 (the increase in the force up to this moment of time is nearly linear in character). Lines 1 and 3 correspond to the maximum forces on the cylinder and the forces with different accelerations of the sphere; the overshooting of the maximum force F_C during sudden motion relative to the steady-state value (line 2) is about 50%.

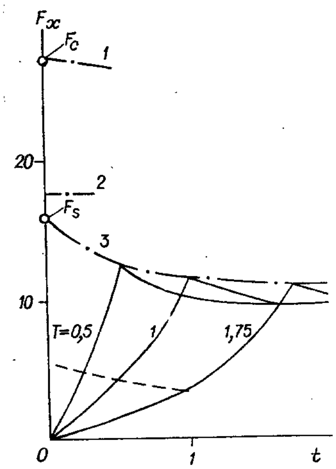

Fig. 1

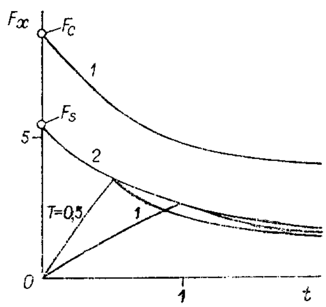

Fig. 2

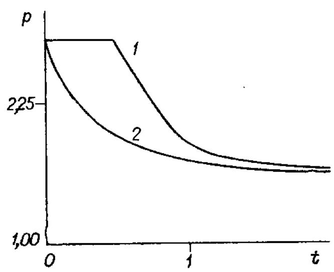

Fig. 3

The dependence of the nonsteady force on time with instantaneous acceleration for the same bodies was also obtained for M = 0.8 (Fig. 2, where curve 1 shows the results for the end of the cylinder and curve 2 shows data for the sphere).

Figure 3 (M = 0.8) shows the dependence of pressure on time at the frontal point of the bodies. Pressure is initially constant for the end of the semiinfinite cylinder (curve 1). After arrival of the perturbations from the edge, the pressure decreases to a value equal to the stagnation pressure at the critical point in steady motion of the body. The characteristic time of the transitional process and the characteristic time of action of the nonsteady force for the end of the cylinder are of the same order of magnitude as for the sphere (line 2, $\Delta t \approx 1$). The maximum pressure for the cylinder is 1.8 times greater than the steady-state pressure, while the maximum pressure for the sphere is 1.65 times greater. It follows from the above analysis that the maximum loads correspond to instantaneous acceleration and develop at the initial moment of time. They can be found by the method described below, which is simpler than numerical calculation of the entire problem.

3. Method of Calculating the Maximum Loads. We will study the case when a body of a specified form suddenly changes from a state of rest to a state of uniform motion with a prescribed Mach number. We formulate the following problem: find the pressure distribution on the surface of the body and the instantaneous force at the initial moment of time, which determine the maximum loads.

We will examine the coordinate system x, y, z connected with the body. We construct the plane z = 0 so that the component of the external normal on z is directed upward at z > 0 and downward at z < 0 (it is assumed that such a plane exists for the bodies being examined; however, this restriction is not essential, and the method can be used for bodies of arbitrary form).

We assign the equation of the surface of the body in the form of two surfaces located in upper and lower half-spaces:

$$z \geqslant 0,\ z = z_+(x,\ y);\ z < 0,\ z = z_-(x,\ y). \tag{3.1}$$

The components of the vector of the external normal $\mathbf{n}$ can be written in the form

$$\begin{gathered}
z \geqslant 0, \quad n_x = \frac{-\dfrac{\partial z_+}{\partial x}}{\sqrt{1+\left(\dfrac{\partial z_+}{\partial x}\right)^2+\left(\dfrac{\partial z_+}{\partial y}\right)^2}}, \quad n_y = \frac{-\dfrac{\partial z_+}{\partial y}}{\sqrt{1+\left(\dfrac{\partial z_+}{\partial x}\right)^2+\left(\dfrac{\partial z_+}{\partial y}\right)^2}},\\
n_z = \frac{-1}{\sqrt{1+\left(\dfrac{\partial z_+}{\partial x}\right)^2+\left(\dfrac{\partial z_+}{\partial y}\right)^2}};\\
z < 0, \quad n_x = \frac{\dfrac{\partial z_-}{\partial x}}{\sqrt{1+\left(\dfrac{\partial z_-}{\partial x}\right)^2+\left(\dfrac{\partial z_-}{\partial y}\right)^2}},\\
n_y = \frac{\dfrac{\partial z_-}{\partial y}}{\sqrt{1+\left(\dfrac{\partial z_-}{\partial x}\right)^2+\left(\dfrac{\partial z_-}{\partial y}\right)^2}}, \quad n_z = \frac{1}{\sqrt{1+\left(\dfrac{\partial z_-}{\partial x}\right)^2+\left(\dfrac{\partial z_-}{\partial y}\right)^2}}.
\end{gathered} \tag{3.2}$$

The velocity vector of the incoming flow, forming the angle α with the x axis, is $\mathbf{V} = \mathbf{i}V_\infty \times \cos\alpha + \mathbf{k}V_\infty \sin\alpha$. We find the projection of velocity on the normal to the body: $V_n = (\mathbf{V},\ \mathbf{n}) = V_\infty n_x \cos\alpha + V_\infty n_z \sin\alpha$. Rarefaction develops on the surface at $V_n > 0$ (we will designate it as σ_-), while compression occurs at $V_n < 0$ (the surface σ^+). The condition $V_n = 0$ gives the interface between the regions of action of forces associated with pressure. In accordance with the unidimensional theory of the decay of an arbitrary discontinuity [1, 4], we obtain the following formulas for the pressure at each point of the surface by solving the problem of the sudden motion of a piston in the direction of the normal to the body and changing over to the coordinate system connected with the body: in the case of compression,

$$p_+ = 1 + \frac{\gamma(\gamma+1)}{4}M_n^2 + \gamma M_n \sqrt{1 + M_n^2\left(\frac{\gamma+1}{4}\right)^2}; \tag{3.3}$$

in the case of rarefaction,

$$p_- = \left(1 - \frac{\gamma - 1}{2} M_n\right)^{2\gamma/(\gamma-1)} \tag{3.4}$$

($M_n = |V_n|/a_\infty$; a_∞ is the speed of sound in the incoming flow).

To find the total force in the direction of the x axis for an arbitrary body, it is necessary to calculate the integrals over the surface by replacing it by small planar elements and passing to the limit:

$$F_x = \iint_{\sigma_+} p_+(M_n) \sin(\mathbf{n}, x)\, d\sigma_+ - \iint_{\sigma_-} p_-(M_n) \sin(\mathbf{n}, x)\, d\sigma_-. \tag{3.5}$$

For a sphere of unit radius (3.5), we have

$$F_x = 2\pi \int_0^{\pi/2} p_+(M_n) \cos\varphi \sin\varphi\, d\varphi - 2\pi \int_0^{\pi/2} p_-(M_n) \cos\varphi \sin\varphi\, d\varphi \quad \left(\varphi = (\mathbf{n}, x) - \frac{\pi}{2}\right). \tag{3.6}$$

Calculation of the first integral in (3.6) leads to the expression

$$F_x^+ = 2\pi \left\{\frac{1}{2} + \frac{\gamma(\gamma+1)}{16} M^2 + \frac{\gamma(\gamma+1)}{4} M^2 \left[\frac{1}{4} \sqrt{\left(1 + \frac{16}{(\gamma+1)^2 M^2}\right)^3} - \frac{2}{(\gamma+1)^2 M^2} \sqrt{1 + \frac{16}{(\gamma+1)^2 M^2}} - \frac{32}{(\gamma+1)^4 M^4} \ln\left(1 + \sqrt{1 + \frac{16}{(\gamma+1)^2 M^2}}\right) + \frac{32}{(\gamma+1)^4 M^4} \ln \frac{4}{(\gamma+1) M}\right]\right\}. \tag{3.7}$$

For the second integral in (3.6) we have

$$F_x^- = \frac{8\pi}{(\gamma-1) M^2} \left\{\frac{1 - \left(1 - \frac{\gamma-1}{2} M\right)^{2\gamma/(\gamma-1)+1}}{3\gamma - 1} - \frac{1 - \left(1 - \frac{\gamma-1}{2} M\right)^{2\gamma/(\gamma-1)+2}}{4\gamma - 2}\right\}. \tag{3.8}$$

Figure 4 shows the results of calculation of the forces from (3.7) and (3.8) for different M ($\gamma = 1.4$). Here, lines 1 and 2 show the values of F_x^+ and F_x^-, the dot-dashed line shows the force $F_x^+ - F_x^-$, and the dashed line shows F_x^+ calculated from the approximate formula

$$F_x^+ = \pi\left(1 + \frac{2}{3}\gamma M + \frac{\gamma(\gamma+1)}{8} M^2\right), \tag{3.9}$$

obtained by expansion of Eq. (3.7) with small M. It can be seen from the graph that Eq. (3.9) satisfactorily describes the force in the range M = 0-1.5. At $\gamma = 1.4$ and M = 2, $F_x^+ = 16.38$ and $F_x^- = 0.5069$; their difference is shown in Fig. 1 by the circle F_s.

For a cylinder of specified length with flat ends at a zero angle of attack, the use of (3.5) gives

$$F_x = \pi\left[1 + \frac{\gamma(\gamma+1)}{4} M_n^2 + \gamma M_n \sqrt{1 + M_n^2 \left(\frac{\gamma+1}{4}\right)^2}\right] - \pi\left[1 - \frac{\gamma-1}{2} M_n\right]^{\frac{2\gamma}{\gamma-1}}.$$

The value of M_n coincides with M. For M = 0.8, the force acting on the front end is equal to 8.72. The force on the rear end in this case is 0.927. The circle F_c in Fig. 2 shows the theoretical value of the force on the front surface.

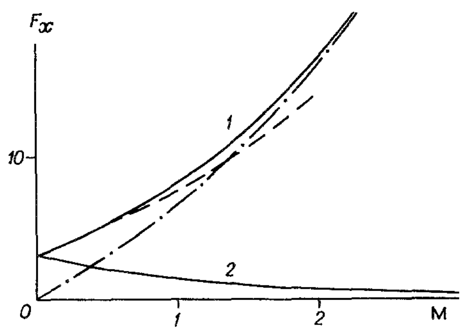

Fig. 4

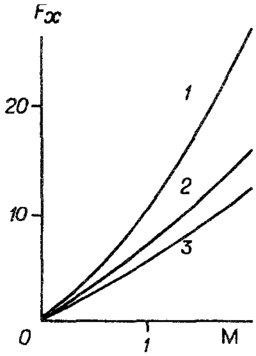

Fig. 5

Using Eqs. (3.1)-(3.5) and the geometric characteristics of the body, we can obtain formulas for the pressure and the total force on bodies of different form, including the case of a nonzero angle of attack.

As an example, let us examine a circular cone of finite length x_0 with a base of radius 1. We direct the x axis along the axis of the cone. Then its surface is described by the equations $z \geq 0$, $z_+ = \sqrt{\bar{x}^2 - y^2}$; $z < 0$, $z_- = -\sqrt{\bar{x}^2 - y^2}$ $(\bar{x} = x/x_0)$. Calculating the derivatives, we determine the unit vector of the external normal

$$z>0,\quad \mathbf{n}_+ = \left\{-\frac{1}{\sqrt{1+x_0^2}},\ \frac{x_0}{\sqrt{1+x_0^2}}\frac{y}{x},\ \frac{x_0}{\sqrt{1+x_0^2}}\sqrt{1-\left(\frac{y}{x}\right)^2}\right\};$$

$$z<0,\ \mathbf{n}_- = \left\{-\frac{1}{\sqrt{1+x_0^2}},\ \frac{x_0}{\sqrt{1+x_0^2}}\frac{y}{x},\ -\frac{x_0}{\sqrt{1+x_0^2}}\sqrt{1-\left(\frac{y}{x}\right)^2}\right\}.$$

Making the substitution $x_0 = \cos\varphi/\sin\varphi$, we finally have

$$z>0,\ \mathbf{n}_+ = \left\{-\sin\varphi,\ \frac{y}{x}\cos\varphi,\ \sqrt{1-\left(\frac{y}{x}\right)^2}\cos\varphi\right\};$$

$$z<0,\ \mathbf{n}_- = \left\{-\sin\varphi,\ \frac{y}{x}\cos\varphi,\ -\sqrt{1-\left(\frac{y}{x}\right)^2}\cos\varphi\right\}.$$

The projection of the velocity vector **V** on the normal **n** for $z > 0$ and $z < 0$ takes the form (the plus sign in the first case)

$$(\mathbf{V},\mathbf{n}) = V_\infty\left[-\cos\alpha\sin\varphi \pm \sin\alpha\cos\varphi\sqrt{1-\left(\frac{y}{x}\right)^2}\right]. \tag{3.10}$$

Let us examine the region of angles of attack $0 \leq \alpha \leq \pi/2$. Given this condition, compression always occurs on the lower surface. On the upper surface, the interface (compression–rarefaction) is determined by the equation $y/x = \pm\sqrt{1 - (\tan\ /\tan\alpha)^2}$. We calculate the pressure at an arbitrary point on a cone at the initial moment of motion by using (3.3), (3.4), and (3.10). In the compression region (the minus sign corresponding to $z < 0$, the plus sign to $z > 0$)

$$\begin{aligned} p_+ = 1 &+ \frac{\gamma(\gamma+1)}{4}\mathrm{M}^2\left[-\cos\alpha\sin\varphi \pm \sin\alpha\cos\varphi\sqrt{1-\left(\frac{y}{x}\right)^2}\right]^2 + \\ &+ \gamma\mathrm{M}\left|-\cos\alpha\sin\varphi \pm \sin\alpha\cos\varphi\sqrt{1-\left(\frac{y}{x}\right)^2}\right| \times \\ &\times\sqrt{1+\mathrm{M}^2\left(\frac{\gamma+1}{4}\right)^2\left[-\cos\alpha\sin\varphi \pm \sin\alpha\cos\varphi\sqrt{1-\left(\frac{y}{x}\right)^2}\right]^2}. \end{aligned} \tag{3.11}$$

The rarefaction region on the cone is formed at $\varphi < \alpha$ and is found from the condition $z > 0$, $|y/x| \leq \sqrt{1 - (\tan\alpha/\tan\alpha)^2}$. The pressure in this region

$$p_{-}=\left[1-\frac{\gamma-1}{2}\mathrm{M}\left|-\cos\alpha\sin\varphi+\sin\alpha\cos\varphi\sqrt{1-\left(\frac{y}{x}\right)^{2}}\right|\right]^{2\gamma/(\gamma-1)}. \qquad (3.12)$$

The rarefaction in the bottom part of the cone is calculated in the same manner as for the end of the cylinder. In accordance with (3.11) and (3.12), the maximum compression and rarefaction on the lateral surface of the cone arises at $y = 0$ on the bottom and top surfaces of the cone, respectively.

Figure 5 shows the theoretical maximum forces acting on a body of finite dimensions (end of a cylinder, sphere, and cone with half divergence angle of 30° – lines 1-3) in relation to the value of M associated with sudden motion at $\alpha = 0$.

The instantaneous loads on bodies of another form can be similarly determined. This includes three-dimensional configurations.

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